FACULTY OF ENGINEERING & ARCHITECURE - AMERICAN UNIVERSITY OF BEIRUT

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ON MY HONOR, I WILL NOT GIVE OR RECEIVE
ANY ASSISTANCE ON THIS QUIZ OR EXAM.

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Type of flow

Laminar flow
Turbulent flow

The Reynolds Number, N_R, relationship is used

$$N_R = \frac{(928)(d)(\rho)v}{\mu}$$

Where N_R = dimensionless number
d = diameter, in
ρ = density, lbm/gal
v = velocity, ft/sec
μ = viscosity, cp

Thru experiment

$$N_R < 2000 \Rightarrow \text{laminar flow}$$
$$N_R > 2000 \Rightarrow \text{Turbulent flow}$$

<u>Newtonian Fluid - Laminar flow - In Pipe</u>

Poiseuille equation states the relationship between Pressure due to friction and other flow factors for a Newtonian fluid under laminar flow Conditions in a straight, circular pipe, or

$$\Delta P_f = \frac{\mu L v}{1500\, d^2}$$

Where ΔP_f = psi, L = ft, d = in, μ = cp, v = ft/sec
d = in

Newtonian fluid - Turbulent flow - In pipe

The Fannin equation states the relationships between pressure drop due to friction and other flow factors for a Newtonian fluid under turbulent flow conditions in a straight, circular pipe, or

$$\Delta P_f = \frac{f \rho L v^2}{25.8\, d}$$

Where f = dimensionless = fannin friction Factor related by experiment to the Reynolds Number, N_R, diameter of the conductor, and the condition of the surface of the inside of the conductor. Can be shown as a function of N_R.

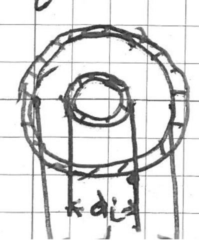
Newtonian fluid - laminar flow - In Annulus

To use the Poiseuille eqn. for laminar in the annulus, the annulus cross-section area must be expressed as an equivalent cross-sectional area of a pipe which will have the same pressure drop / length of the same flow rate.

Equivalent diameter of the annulus, de, is defined as 4 times the hydraulic radius, and the hydraulic radius is defined as

$$r_h = \frac{\text{Cross-sectional area of flow}}{\text{Wetted Parameter}}$$

Where, the wetted perimeter is the total length of surface contacted by the field or



d_o = inside diameter of the outside conductor
d_i = outside diameter of the inside conductor

$\therefore$ $$de = 4r_h = \frac{(4)(\pi/4)(d_o^2 - d_i^2)}{\pi(d_o + d_i)} = (d_o - d_i)$$

the actual velocity, v_{act}, in an annulus is,

$$v_{act} = \frac{q}{2.45(d_o^2 - d_i^2)} \quad ft/sec$$

Type of flow in an annulus

Calculate the equivalent Reynolds Number using the equivalent diameter and the actual velocity or

$$N_{re} = \frac{757(d_o - d_i)\rho v_{act}}{\mu} \Rightarrow N_{re} < 2000, \text{ laminar flow}$$

$$N_{re} = \frac{757 \rho \, de \, v_{act}}{\mu} \Rightarrow N_{re} > 2000, \text{ turbulent flow}$$

If laminar use the Poiseuille equation modified for annular flow to calculate pressure drop, or

$$\Delta P_f = \frac{\mu L v_{act}}{1000(d_o - d_i)^2}$$

Newtonian fluid - Turbulent flow - in Annulus

$$de = d_o - d_i$$

Type of flow in an annulus

$$N_{re} = \frac{757 \, de \, \rho v_{act}}{\mu} \Rightarrow N_{re} < 2000 \Rightarrow \text{laminar flow}$$

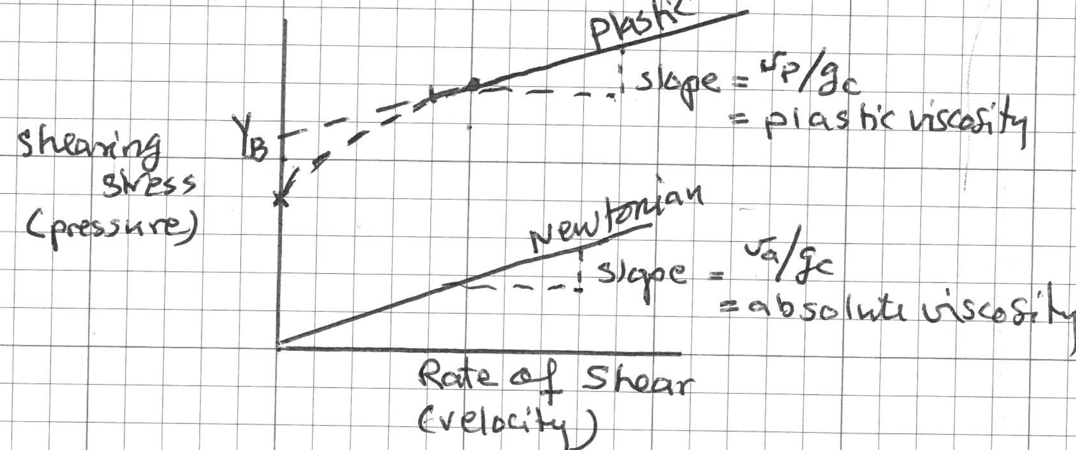
$$N_{re} > 2000 \Rightarrow \text{turbulent flow}$$

If turbulent, use the Fannin equation to calculate pressure drop

$$\Delta P_f = \frac{f L v_{act}^2}{25.8 \, de}$$

Plastic Fluid-Laminar flow - in Pipe

Referring to the plastic fluid viscosity curve shown below



The eqn. for the straight line portion of the curve is

$$\Delta P_f = m v + Y_B$$

and the Poiseuille eqn. for laminar flow is,

$$\Delta P_f = \frac{32 \mu v L}{g_c d^2}$$

or

$$\text{Slope} = m = \frac{\Delta P_f}{v} = \frac{32 \mu L}{g_c d^2}$$

then,
$$\Delta P_f = \frac{(32 \mu L)(v)}{g_c d^2} + Y_B$$

and

Y_B expressed in equivalent pressure terms $= \dfrac{4 Y_B L}{d}$

then
$$\Delta P_f = \frac{32 \mu_p L v}{g_c d^2} + \frac{4 Y_B L}{d}$$

or
$$\Delta P_f = \frac{\mu_p L v}{500 d^2} + \frac{Y_B L}{300 d} \quad \text{(Field units)}$$

If these equations are to be used for a plastic fluid, an

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equivalent viscosity, μ_e, which is the viscosity a plastic fluid would have if it were a Newtonian fluid, must be used, and

$$\mu_e = \frac{5 Y_B d}{v} + \mu_p$$

Since μ_e is an equivalent Newtonian viscosity, it can be used in Reynolds Number equation,

or,

$$N_R = \frac{928\, d\, \rho\, v}{\frac{5 Y_B d}{v} + \mu_p} \quad ; \; N_R < 2000, \text{ laminar} \quad ; \; N_R > 2000, \text{ turbulent}$$

Since $N_r = 2000$ and solving for velocity yields a critical velocity, v_c, and an actual velocity below which is laminar flow and an actual velocity above which is turbulent flow,

or $v_{act} < v_c$, laminar flow

$v_{act} > v_c$, turbulent flow

then,

$$N_R = 2000 = \frac{928\, d\, \rho\, v}{\frac{5 Y_B d}{v} + \mu_p}$$

or,

$$\boxed{v_c = \frac{1.08\,\mu_p + 1.08\left(\mu_p^2 + 9.3\,\rho\, d^2\, Y_B\right)^{0.5}}{\rho\, d}} \quad \text{(field units)}$$

and $v_{act} = \dfrac{q}{2.45\, d^2}$ (field units)

Plastic Fluid - Laminar flow - in Annulus

The poiseuille eqn. for laminar flow applies only to a straight, circular pipe and cannot be used directly if the cross-section area is an annulus. To use this eqn. for an annulus, the annular cross-sectional area must be expressed as an equivalent cross-sectional area of pipe which will have the same pressure drop - flow rate relation as the equivalent diameter of the annulus, or, equivalent diameter, d_e, is defined as

$$r_h = \frac{\text{Cross-sectional area of flow}}{\text{wetted diameter}}$$

where the wetted diameter is the total length of surface contacted by the fluid,

d_o = inside diameter of the outside conductor

d_i = outside diameter of the inside conductor

$$d_e = 4r_h = \frac{(4)(\pi/4)(d_o^2-d_i^2)}{\pi(d_o+d_i)}$$

$$= d_o - d_i$$

The actual velocity, v_{act} in an annulus is,

$$v_{act} = \frac{q}{2.45(d_o^2-d_i^2)} \quad ft/sec$$

Type of Flow in an Annulus

The expression for determining the critical velocity, v_c, is modified when the flow is in annulus, or, Date :

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$$v_c = \frac{1.08\,\mu_p + 1.08\left(\mu_p^2 + 6.98\,\rho\, d_e^2\, Y_B\right)^{0.5}}{\rho\, d_e}$$

where $d_e = d_o - d_i$

Determining if the flow is laminar or turbulent by calculating the critical velocity, v_c, using the equivalent diameter, $(d_o - d_i)$ and comparing to the actual velocity, v_{act},

$v_{act} < v_c \Rightarrow$ Laminar flow

$v_{act} > v_c \Rightarrow$ turbulent flow

If laminar, the Poiseuille eqn. modified for laminar flow must be used to calculate ΔP_f, using d_e, v_{act} and μ_p

$$\Delta P_f = \frac{\mu_p L\, v_{act}}{1000(d_o - d_i)^2} + \frac{Y_B L}{267(d_o - d_i)}$$

Plastic fluid - Turbulent flow - in Annulus

$d_e = d_o - d_i$

Type of flow in an Annulus

$v_{act} < v_c \Rightarrow$ laminar

$v_{act} > v_c \Rightarrow$ turbulent

If flow is turbulent, the Fannin eqn. must be used to calculate ΔP_f in a straight circular pipe.

<u>Note:</u> (Viscosity does not appear in the Fannin eqn. for turbulent flow except in determining the friction factor, f, from the Reynolds Number eqn. and f – Curves and has little effect on the ΔP_f calculation when turbulent flow exists).

Determining f using the f-Curves and N_R, calculated using the $\bar{v}_{act}$ and plastic viscosity, μ_p, and equivalent diameter, de,

$$N_R = \frac{928\, d_e \bar{v}_{act}}{\mu_p}$$

then
$$\Delta P_f = \frac{f \rho L v_{act}^2}{25.8\, d_e}$$